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#define MAPM_NUM_DEC_PLC_OPN 240
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```
#define MAPM_NUM_DEC_PLC_SHOW 240
```

```
/* -----  
CDPS2_sqrt2_doac_2nd_order_verif_arb_prec  
-----
```

Function: sqrt(2) difference-of-alternating-coefficients (DOAC), 2nd-order recurrence relation used to generate denominators, arbitrary precision

Description: allow the user to 'verify' numerically that the sqrt(2) series representation using the DOAC approach and the 2nd-order recurrence relation for generating denominators is correct. As more terms in the series are generated, the approximation of $\sqrt{2}^2$ converges to 2.

Usage: the function allows the user to specify whether the series representation will converge to sqrt(2) from above or below.

Implementation notes:

- uses Mike's Arbitrary Precision Math (MAPM) C-language library
- I typically first implement a function as 32-bit (not arbitrary precision) to verify the algorithm and then convert it to arbitrary precision. The comments

in the code reflect this approach. The arbitrary precision code is usually much harder to read / understand than the 32-bit version.

- the comments are from the 'from above' version of the algorithm

```
----- */
```

```
int CDPS2_sqrt2_doac_2nd_order_verif_arb_prec(
```

```
    int    from_above) { // flag  
    int    num_iter;     // number of iterations  
    int    i;            // counter  
    M_APM  d0_ap = (void *)0;  
    M_APM  d1_ap = (void *)0;  
    M_APM  d2_ap = (void *)0;  
    M_APM  C2_ap = (void *)0;  
    M_APM  C3_ap = (void *)0;  
    M_APM  C24_ap = (void *)0;  
    M_APM  C34_ap = (void *)0;  
    M_APM  t0_ap = (void *)0;  
    M_APM  t1_ap = (void *)0;  
    M_APM  s_ap = (void *)0;  
    char   *d_ap_str = (void *)0;  
    char   *s_ap_str = (void *)0;
```

```
    m_apm_cpp_precision(MAPM_NUM_DEC_PLC_OPN);
```

```
        // -- allocate arbitrary precision variables --
```

```
    d0_ap = m_apm_init();  
    d1_ap = m_apm_init();  
    d2_ap = m_apm_init();  
    C2_ap = m_apm_init();  
    C3_ap = m_apm_init();
```

```

C24_ap = m_apm_init();
C34_ap = m_apm_init();
t0_ap = m_apm_init();
t1_ap = m_apm_init();
s_ap = m_apm_init();

// -- initialize variables --
m_apm_set_long(C2_ap,2);
m_apm_set_long(C3_ap,3);
m_apm_set_long(C34_ap,34);

if (!from_above) {
    m_apm_set_long(C24_ap,-24);
    m_apm_set_long(d0_ap,5);
    m_apm_set_long(d1_ap,145); }
else {
    m_apm_set_long(C24_ap,24);
    m_apm_set_long(d0_ap,24);
    m_apm_set_long(d1_ap,840); }

// -- query user --
printf("Number of iterations? ");
scanf("%d",&num_iter);

// -- generate the first few terms before entering
// loop --
printf("\nsqrt(2) =\n");

if (!from_above)
    printf("\n 1 / 1 +\n");
else
    printf("\n 3 / 2 -\n");

// s = 1.5;

if (!from_above)
    m_apm_set_double(s_ap,1.0);
else
    m_apm_divide(s_ap,MAPM_NUM_DEC_PLC_OPN,C3_ap,C2_ap);

// printf("\n %d / %d -\n",2,d0);

d_ap_str = m_apm_to_fixpt_stringexp(0,d0_ap,'.',' ',3);
if (!from_above)
    printf("\n %d / %s +\n",2,d_ap_str);
else
    printf("\n %d / %s -\n",2,d_ap_str);
free(d_ap_str);

// s -= (double)2/d0;

```

```

m_apm_divide(t0_ap,MAPM_NUM_DEC_PLC_OPN,C2_ap,d0_ap);
if (!from_above)
    m_apm_add(t1_ap,s_ap,t0_ap);
else
    m_apm_subtract(t1_ap,s_ap,t0_ap);
m_apm_copy(s_ap,t1_ap);

// printf("\n    %d / %d -\n",2,d1);

d_ap_str = m_apm_to_fixpt_stringexp(0,d1_ap,'.',' ',3);
if (!from_above)
    printf("\n    %d / %s +\n",2,d_ap_str);
else
    printf("\n    %d / %s -\n",2,d_ap_str);
free(d_ap_str);

// s -= (double)2/d1;

m_apm_divide(t0_ap,MAPM_NUM_DEC_PLC_OPN,C2_ap,d1_ap);
if (!from_above)
    m_apm_add(t1_ap,s_ap,t0_ap);
else
    m_apm_subtract(t1_ap,s_ap,t0_ap);
m_apm_copy(s_ap,t1_ap);

                // -- loop --
for (i=0; i<num_iter; i++) {

//    d2 = C24 + 34*d1 - d0

    m_apm_multiply(t0_ap,d1_ap,C34_ap);
    m_apm_subtract(t1_ap,t0_ap,d0_ap);
    m_apm_add(d2_ap,t1_ap,C24_ap);

//    printf("\n    %d / %d -\n",2,d2);

    d_ap_str = m_apm_to_fixpt_stringexp(0,d2_ap,'.',' ',3);
    if (!from_above)
        printf("\n    %d / %s +\n",2,d_ap_str);
    else
        printf("\n    %d / %s -\n",2,d_ap_str);
    free(d_ap_str);

//    s -= (double)2/d2;

    m_apm_divide(t0_ap,MAPM_NUM_DEC_PLC_OPN,C2_ap,d2_ap);
    if (!from_above)
        m_apm_add(t1_ap,s_ap,t0_ap);
    else
        m_apm_subtract(t1_ap,s_ap,t0_ap);
    m_apm_copy(s_ap,t1_ap);

```

```

/*
    d0 = d1;
    d1 = d2;
*/
    m_apm_copy(d0_ap,d1_ap);
    m_apm_copy(d1_ap,d2_ap); }

    printf("\n    ...\n");

// printf("\nsqrt(2) = %30.271f\n",s);

    s_ap_str = m_apm_to_fixpt_stringexp(MAPM_NUM_DEC_PLC_SHOW,s_ap,'.',' ',3);
    printf("\nsqrt(2) = %s\n",s_ap_str);
    free(s_ap_str);

// printf("\nsqrt(2)^2 = %30.271f\n",s*s);

    m_apm_multiply(t0_ap,s_ap,s_ap);
    m_apm_copy(s_ap,t0_ap);

    s_ap_str = m_apm_to_fixpt_stringexp(MAPM_NUM_DEC_PLC_SHOW,s_ap,'.',' ',3);
    printf("\nsqrt(2)^2 = %s\n",s_ap_str);
    free(s_ap_str);

    printf("\n");

    // -- free the variables --
    m_apm_free(d0_ap);
    m_apm_free(d1_ap);
    m_apm_free(d2_ap);
    m_apm_free(C2_ap);
    m_apm_free(C3_ap);
    m_apm_free(C24_ap);
    m_apm_free(C34_ap);
    m_apm_free(t0_ap);
    m_apm_free(t1_ap);
    m_apm_free(s_ap);

    return (TRUE); }

```